# Technical Notes

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## Three-Dimensional Asymptotic Suction of Compressible Flow

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### Introduction

THE use of wall suction to increase lift and reduce the drag of airfoils is a subject of increasing practical importance. Solutions to the two-dimensional boundary-layer equations are available for several geometries, suction velocity distributions, and compressible, as well as incompressible flow.[1] The problem of three-dimensional flow with suction is the one that remains to be solved. Liu[2] extended the case of asymptotic suction flow along a flat plate to provide a solution of asymptotic suction flow near a corner. Recently, Maddaus and Shanebrook[3] obtained a series solution of the three-dimensional boundary-layer equation under the condition of asymptotic suction. There are only a few solutions published for the case of compressible flow. Young[4] showed that an asymptotic solution also exists for the case of compressible flow along a flat plate in the presence of uniform suction.

Although many numerical techniques are available to compute the three-dimensional laminar boundary-layer development, it is of value to have some exact solutions in order to evaluate the numerical results. This Note is intended to present a solution to the Navier-Stokes equations for the flow of a compressible fluid near a corner when the asymptotic suction condition is reached. The velocity and the temperature profiles at the center symmetric plane of the corner are presented. The results may be used as a test case for three-dimensional viscous flow computer codes.

### Formulation and Solution

We consider the steady flow of compressible fluid near a corner formed by two perpendicular flat plates at zero incidence with uniform suction. At large distance from the leading edge the asymptotic suction condition can be reached and the properties of the flow are independent of longitudinal distance. Liu and Ismail[5] demonstrated that, under this condition, the velocity components normal to the plates are constant throughout the flowfield in the asymptotic region. For the case of compressible fluid the same argument can be applied. From the continuity equation, we have

$$\rho v = \rho w = \text{const} = \rho_\infty v_\infty \tag{1}$$

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Equation (1) shows that the surface velocity is not normal to the surface. It is not possible in the case of solid wall, but with a uniformly distributed porous wall a small slip velocity may be allowed. If the holes are inclined to the surface, the fluid may pass through the surface with the appropriate velocity components. Then, the momentum and the thermal energy equations become

$$\rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y} + \frac{\partial}{\partial z} \mu \frac{\partial u}{\partial z} \tag{2}$$

$$\rho C_p \left( v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \mu \left( \frac{\partial u}{\partial z} \right)^2 \tag{3}$$

where $u$, $v$, and $w$ are the velocity components in $x$, $y$, and $z$ directions, respectively; $x$ is measured from the leading edge along the corner; $y$ and $z$ are coordinates normal to the plates, $\rho$ the density of the fluid, $\mu$ dynamic viscosity, $C_p$ the specific heat, $T$ the temperature, and $k$ the conductivity of the fluid.

The boundary conditions are

$$u(x,0,z) = 0 \qquad u(x,y,0) = 0 \qquad u(x,\infty,\infty) = U$$

$$T(x,0,z) = T_a \qquad T(x,y,0) = T_a \qquad T(x,\infty,\infty) = T_\infty \tag{4}$$

where $U$ is the freestream velocity, $T_\infty$ the freestream temperature, $T_a$ the adiabatic temperature at the wall, and $v_\infty$ the velocity in the $y$ direction at freestream which is equal to $w_\infty$.

Changing the independent and dependent variables by

$$y_I = -\frac{v_\infty}{\nu_\infty} \int_0^y \frac{\rho}{\rho_\infty} \mathrm{d}y \quad z_I = -\frac{v_\infty}{\nu_\infty} \int_0^z \frac{\rho}{\rho_\infty} \mathrm{d}z \quad u_I = \frac{u}{U} \tag{5}$$

and assuming the viscosity is proportional to the temperature, Eq. (2) can be transformed to

$$-\frac{\partial u_I}{\partial y_I} - \frac{\partial u_I}{\partial z_I} = \frac{\partial^2 u_I}{\partial y_I^2} + \frac{\partial^2 u_I}{\partial z_I^2} \tag{6}$$

Equation (6) and the boundary conditions are exactly the same forms as that of incompressible flow.[2] For two-dimensional flow, Eq. (6) becomes the same form as that presented by Young.[4] The solution of Eq. (6) to satisfy the boundary conditions is

$$u_I 1 \ (1 - e^{-y_I})(1 - e^{-z_I}) \tag{7}$$

Following Crocco[1] we assume that the temperature is a function of velocity ($T = T(u)$), Eq. (3) can be transformed to

$$C_p \frac{P_r - 1}{p} \left[ T_u \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y} + T_u \frac{\partial}{\partial z} \mu \frac{\partial u}{\partial y} \right]$$

$$= (T_{uu} k + \mu) \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] \tag{8}$$

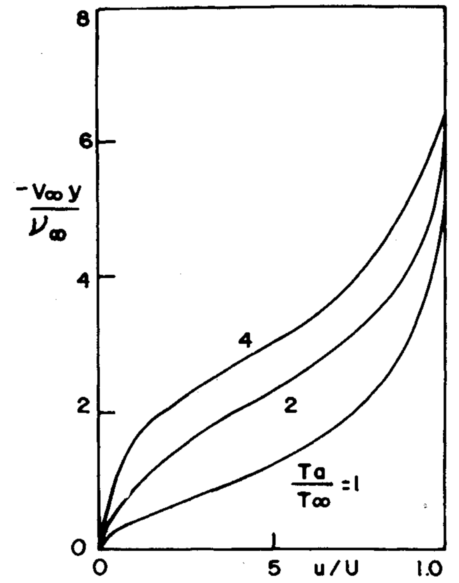


Fig. 1 Velocity profiles at the center symmetric plane.

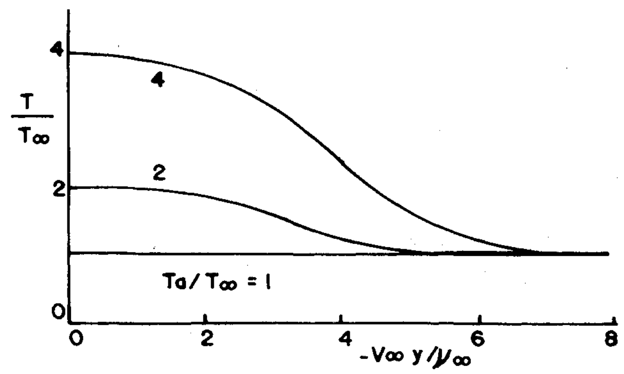


Fig. 2 Temperature profiles at the center symmetric plane.

where $Pr$ is the Prandtl number, $T_u = \mathrm{d}T/\mathrm{d}u$, and $T_{uu} = \mathrm{d}^2T/\mathrm{d}u^2$. When $Pr = 1$, we have

$$T_{uu} = -(1/C_p) \tag{9}$$

which is the same form as that for two-dimensional flow. Integrating Eq. (9) twice and with the aid of Eq. (5), we have the temperature distribution

$$\frac{T}{T_\infty} = 1 + \left(\frac{T_a}{T_\infty} - 1\right)\left(1 - \frac{u^2}{U^2}\right) \tag{10}$$

Again, it is the same form as that for incompressible flow.

The velocity profile cannot be plotted easily in terms of real coordinates $y$ and $z$, because both $y_l$ and $z_l$ depend on the temperature and the temperature depends on the velocity. Also, an explicit expression cannot be obtained as in the case of two-dimensional flow. Thus, the profiles were obtained by the iteration approach. The incompressible velocity profile is used as the initial profile to calculate the temperature. Then, the new velocity is calculated in terms of the real coordinates. Figure 1 shows the velocity profile at the center symmetric plane for the values of $T_a/T_\infty$. Figure 2 shows the temperature distribution. The incompressible case is also shown. It can be observed that there is a thickening of the velocity and temperature boundary layer with increase of velocity, which agrees with the two-dimensional case.

## References

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# Nonreflecting Boundary Conditions for the Complete Unsteady Transonic Small-Disturbance Equation

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## Introduction

ONE of the most widely used programs for transonic unsteady aerodynamic analysis is the LTRAN2 code of Ballhaus and Goorjian.[1] That code is used to solve the low-frequency, transonic small-disturbance (TSD) equation

$$A\phi_{xt} = B\phi_{xx} + \phi_{yy} \tag{1}$$

where $\phi$ is a disturbance velocity potential normalized by $cU\delta^{2/3}$, $c$ the airfoil chord, $\delta$ the airfoil thickness ratio, and $U$ the freestream speed. The coefficient $A = 2kM_\infty^2/\delta^{2/3}$, where for an oscillation frequency $\omega$, $k = \omega c/U$, and $M_\infty$ is the freestream Mach number. The coefficient

$$B = \frac{1 - M_\infty^2}{\delta^{2/3}} - M_\infty^2(\gamma + 1)\phi_x$$

where $\gamma$ is the ratio of specific heats. The spatial coordinates $x$ and $y$ and time $t$ are normalized by $c$, $c/\delta^{1/3}$, and $\omega^{-1}$, respectively. Steady-state boundary conditions are used at the airfoil, in the wake, and in the far field.

Houwink and van der Vooren[2] improved LTRAN2 by adding unsteady terms to the airfoil and wake boundary conditions; the resulting code is LTRAN2-NLR. Hessenius and Goorjian[3] added a time derivative term to the downstream condition, as well as unsteady airfoil and wake conditions. Their code, LTRAN2-HI, has been validated in the transonic range by a series of comparisons with experimental data.

The steady-state far-field conditions cause disturbances incident on the computational boundaries to be reflected back into the computational domain. As a result, in LTRAN2 and its variations, the boundaries are placed far enough in the far field such that reflected disturbances do not reach the airfoil

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